Zhu on elementary math -15

# Solving quadratic equations -1

EOM

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《*This article is suitable for students in fifth grade and above and their parents, primary school teachers, middle school teachers, young adults, and any mathematics educator who wants to learn elementary mathematics again.*

After you understand algebraic expressions, algebraic operations and can solve linear equations (can understand the four arithmetic operations and exponential operations, can multiply polynomials, and remember the properties of equality), see, for example, "The First Sweetness of Algebra Learning---Solving Equations", you will find: you can solve quadratic equations of one variable!

For example, let's solve (x-2)(x+3)=0. Using the properties of equality, we reason: the multiplication of two numbers A and B is equal to 0, then either A=0; or B=0 (Think about the reasons for yourself. You can refer to the properties of equality listed later in the article) . So, from (x-2)(x+3)=0, we know: either x-2=0, so x=2; or x+3=0, so x=-3. The conclusion is that the quadratic equation (x-2)(x+3)=0 has two solutions: x=2 or x=-3.

Do a simple algebra calculation:

$$(x-2)(x+3)=x^2+x-6.$$

So, we learn how to solve the following quadratic equation:

$$x^2+x-6=0.$$

To do so, the first step is to write a quadratic polynomial of one variable as the multiplication of two linear polynomials (this process is called factorization):

$$x^2+x-6=(x-2)(x+3).$$

Then use one of the following extended properties of equality:

$$AB = 0 \iff A = 0, \quad \text{or} \quad B = 0.$$

We then can complete solving the equation.

Let's look at another example.

**Example**: Solving Equations

$$x^2 + 8x + 16 = 1\,.$$

Solution: (Don't rush to factorize yet: because the right-hand end is not 0) The original equation is equivalent to

$$x^2 + 8x + 15 = 0\,.$$

First do factorization to get the equivalent equation

$$(\mathrm{x} + 3)(\mathrm{x} + 5) = 0\,.$$

From the above we get x=-3 or x=-5. （If you want to be sure, you can check the results).

It can be seen that as long as you can factorize, solving a quadratic equation is pretty easy .

In fact, it can be proven that if a quadratic equation whose coefficients and constant terms are both rational numbers does have two rational solutions, then we can always factor the quadratic polynomial in the rational number set. The most common factorization method is the method of cross multiplication. If you don't know, ask your parents/teachers, or read a book or search online. Of course, doing factorization is not an easy skill to master and requires some training. You may try this: how long does it take you to determine whether 5609 is a composite number or a prime number.

In the next part we will answer the following questions: what if the equation has no rational solution? For example, how to solve

$$x^2 + 8x + 14 = 0?$$

For the convenience of readers, I list the properties of equality in the following:

*Properties of equality*

$(1)((Invariant\ under\ addition)$ $For\ any\ number\ C,$

$$A = B \iff A + C = B + C.$$

$(2)\ (Invariant\ under\ multiplication)\colon\ For\ any\ C \neq 0,$

$$A = B \iff AC = BC.$$

$(3)\ (Symmetric\ property)\}\colon$

$$A = B \iff B = A.$$

*And*

$(4)(Transitive\ property)$ $If\ A = B,\ and\ B = C,\ then$

$A = C.$